Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 2	DAILY TUTORIAL SHEET 9

171.(ABD)

(A) He can reach (–2) in 2 steps directly in 1 way.
Or he can reach (–2) in 4 steps. For this to happen, he must take a right step within the first 2 steps so that he does not reach (–2) first. $\Rightarrow$ ${}^2C_1$ ways.
So, total ways = 3.

(B) He can reach 3 directly in 1 way in 3 steps. He can reach 3 also in 5 steps. For this to happen, he must take a left step within the first 3 steps. $\Rightarrow$ ${}^3C_1$ ways.
Total ways = 4.

(C) Total ways of taking 5 steps without any constraints = 2^5.
However, we have to subtract scenarios where (–2) or 3 are reached before 5 steps.
He reaches, (–2) in steps: 2^3 such cases.
He reaches (–2) in 4 steps: 2×2 such cases.
He reaches 3 in 3 steps: 1×2^2 such cases.
Hence, totally ways of taking exactly 5 steps $= 2^5 - (2^3 + 2^3) = 16$.

(D) Total ways of performing the experiment $= 16 + 4 + 3 - 3$ [Ways of reaching 3 in 5 steps have been counted twice] $= 20$

172.(AD) $x + y + z + w = 19$
Number of positive integer solutions = number of whole number solutions of the equation $x_1 + y_1 + z_1 + w_1 = 15$
= Number of ways of distributing 15 identical objects to 4 persons ${}^{15+4-1}C_{4-1} = {}^{18}C_3$
= Coefficient of x^{19} in $\left(x + x^2 + x^3 + \ldots + x^{19}\right)^4$

173.(AB) Number of non-negative integer solutions of $x + y + z + w = 10 = {}^{13}C_3 = {}^{13}C_{10}$
Number of ways of distributing 10 identical objects in four different boxes. $\Rightarrow$ (A) is correct option.
Also, it is the same as number of selections of 10 objects from a lot containing 4 varieties of objects
$\Rightarrow$ (B) is correct option ; Clearly (C) and (D) are incorrect.

174.(ABC) If there have to be no same neighbours it implies we have to count clockwise and anticlockwise as same which is $\frac{\text{total}}{2}$. So option (*A*) is correct.
Options (*B*), (*C*) are correct as in garland and necklace also, we consider clockwise and anticlockwise arrangements as same.

175.(BCD) No. of ways = $3^5 - {}^3C_1\, 2^5 + {}^3C_2 = 150$

(A) No. of ways = ${}^5P_3 = 60$ **(B)** No. of parallelograms = ${}^6C_2 \times {}^5C_2 = 150$

(C) No. of ways = $3^5 - {}^3C_1 2^5 + {}^3C_2 = 150$ **(D)** No. of ways = $3^5 - {}^3C_1 2^5 + {}^3C_2 = 150$

176.(ACD) No. of ways $= \frac{\lfloor 6}{\lfloor 2 \lfloor 2 \lfloor 2}$

(A) No. of ways = ${}^{2+3-1}C_{3-1} \times {}^{4+3-1}C_{3-1} = {}^4C_2 \times {}^6C_2 = \frac{\lfloor 4}{\lfloor 2 \lfloor 2} \times \frac{\lfloor 6}{\lfloor 4 \lfloor 2} = \frac{\lfloor 6}{\lfloor 2 \lfloor 2 \lfloor 2}$

(B) No. of ways = $\frac{\lfloor 6}{\lfloor 2 \lfloor 2 \lfloor 2} \times \frac{1}{\lfloor 3}$ **(C)** Coeff. of $x^2\, y^2\, z^2 = \frac{\lfloor 6}{\lfloor 2 \lfloor 2 \lfloor 2}$ **(D)** No. of ways = $\frac{\lfloor 6}{\lfloor 2 \lfloor 2 \lfloor 2}$

177.(ABC) $\frac{(200)!}{\underbrace{2!2!.....2!}_{100\text{ times}}(100)!} = \frac{(200)!}{100!2^{100}} = 1\times3\times5...199$ Also, $\frac{(200)!}{100!2^{100}} = \left(\frac{101}{2}\right)\left(\frac{102}{2}\right)...\left(\frac{200}{2}\right)$

178.(ABC) GHINT first word
Rank of NIGHT = $4!\times3+3!\times2+1=85$
Rank of THING = $4!\times4+3!+2!+1+1=106$
Number of words in between is 20

(A) Number of zeroes at the end of 20! = $\left[\frac{20}{5}\right]$ **(B)** Number of divisors of 20 = $2^2\times5$ is : $3\times2=6$

(C) Number of solutions of $x+y\le19,\ x, y\ge1$ is ${}^{19}C_2 = 171$.

(D) Number of words in dictionary = $5! = 120$.

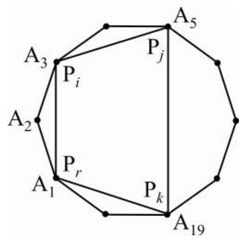
179.(ABC) Total number of ways – Number of ways when R and W are together $= 8! - 7!\,2!$
$= 7!(8-2) = 6(7!) \Rightarrow$ (B)
Now, arrange 6 balls excluding R and W in 6! ways and in 7 gaps, 2 balls R and W can be arranged in ${}^7P_2$ ways. $\Rightarrow$ Total number of ways = ${}^7P_2\times6!$ ways = $2.6!.\,{}^7C_2$ $\Rightarrow$ (C)

180.(BC) The number of ways of inviting, with the couple not included, is ${}^8C_5$. The number of ways of inviting with the couple included, is ${}^8C_3$. Therefore the required number of ways is ${}^8C_5+{}^8C_3 = {}^8C_3+{}^8C_3$ $\left(\because {}^8C_5={}^8C_3\right)$

Also, ${}^{10}C_5 - 2\times{}^8C_4$ because total unconstrained selections = ${}^{10}C_5$

And total bad selections [where exactly one among the couple is selected] = ${}^2C_1\times{}^8C_4$

181.(ABC) Let x, y, z, u be the number of vertices left between any two chosen vertices.
$\Rightarrow\ x, y, z, u\ge1$ and $x+y+z+u=16$ **.... (i)**
Equivalently, $x_1+y_1+z_1+u_1=12;\ x_1, y_1, z_1, u_1\ge0$



Required ways $= \frac{20\times{}^{15}C_3}{4} = \frac{5\times15\times14\times13}{3\times2} = 2275$
Let the 20 vertices $A_1, A_2, ... A_{20}$ be placed in a row.
Let the 4 selected vertices be V_1, V_2, V_3, V_4.
Let x_0 be the number of vertices to left of V_1.
Let x_i be the number of vertices to right of V_i where $i\in\{1, 2, 3, 4\}$.

Now $\sum_{i=0}^{4} x_i = 16$

Also: $x_i\ge1\quad\forall i\in\{1, 2, 3\}$

Number of solutions to above equation is ${}^{17}C_4$.
However, we must eliminate the case $x_0 = x_4 = 0$.
Number of such cases $= {}^{15}C_2$. Hence, ${}^{17}C_4 - {}^{15}C_2$

182.(AC) Total hand-shakes possible $= {}^{2n}C_2$. This will include n hand-shakes in which a person shake hand with their spouse. $\Rightarrow$ Required number $= {}^{2n}C_2 - n = 2n(2n-1)$

183.(AB) Number of times the teacher visits the zoological garden = ${}^{25}C_5$.

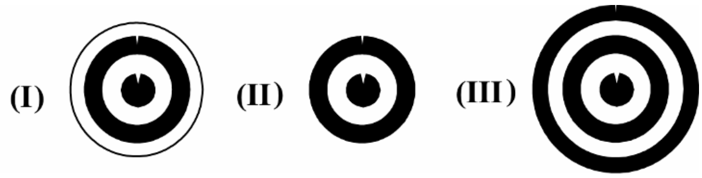
Number of times each child visits the zoo $= {}^{24}C_4$.

∴ Number of times the teacher visits the zoo exceeds the number of times each child visits $= {}^{25}C_5 - {}^{24}C_4$

∴ By Pascal's rule ${}^{24}C_4 + {}^{24}C_5 = {}^{25}C_5$

$\Rightarrow \quad {}^{25}C_5 - {}^{24}C_4 = {}^{24}C_5 \qquad \Rightarrow$ (A) and (B) are correct options.

184.(AC)



There are 12 shots and 12 rings (combined) on 3 targets. Divide 12 shots among 12 places in $\underline{|12}$ ways. But shots on target one has to be in a predefined order (inside to outside) and same is predefined for other two targets. Hence total number of ways $= \dfrac{\underline{|12}}{\underline{|4}\,\underline{|3}\,\underline{|5}}$ or ${}^{12}C_4\ {}^{8}C_3\ {}^{5}C_5$

185.(CD) P R M T T N E U A I O

Since no vowel is between two consonants, we need to keep all consonants together. Considering P R M T T N as 1 packet, there are E, U, A, I, O, P R M T T N i.e. 6 packets which are all different. They are arranged in $\underline{|6}$ ways. Letters of PRMTTN are arranged among themselves in $\dfrac{\underline{|6}}{\underline{|2}}$ ways.

So, by FPC, total $= \underline{|6} \times \dfrac{\underline{|6}}{\underline{|2}} = {}^{6}C_4 \times \underline{|4} \times \underline{|6}$